Macroscopic Traffic Flow Models – A review

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ABSTRACT
In this paper, a comprehensive review of the macroscopic traffic flow models is carried out. A macroscopic traffic flow model is a function which relates salient properties of traffic like speed, flow, and density besides driver’s behaviour. It provides a better understanding of traffic dynamics by predicting the evolution of traffic along a stretch of road with a given set of initial conditions and boundary conditions. An attempt to model traffic flow using kinematic wave theory was initiated by Lighthill and Whitham (1955) and Richards (1956). Later various extensions of this model were proposed; most of them overcame deficiencies in their previous models and made them more realistic. Consequently higher order models were developed and multiple classes were introduced. This not only made models more realistic but also regenerated heterogeneity of traffic flow observed in field. Among the models discussed, speed gradient model, which is a multi-class higher order model, gave a very good representation of heterogeneity without any deficiencies attributed to the higher order models. But none of these models explained the lack of -lane discipline observed in the heterogeneous traffic prevailing in the developing countries like India.
1. **INTRODUCTION**

A traffic flow model is an abstract of traffic flow phenomenon. It represents the salient properties of traffic flow like driver’s behavior, vehicle type and interaction with other vehicles. Modelling could be done on a sub-microscopic scale, microscopic scale, mesoscopic scale or a macroscopic scale, but a macroscopic scale modeling is more preferred due to the fewer amounts of details involved. The parameters used in macroscopic modeling are density, flow and speed. In macroscopic modeling we try to predict the future characteristics of traffic along a stretch of road for small time increments. It is done by representing the propagation of model parameters in form of partial differential equations. It is very helpful in understanding the formation and propagation of traffic congestion. Hence, traffic researchers use traffic flow models to identify possible bottlenecks. It can also be used to simulate the conditions after implementation of different strategies for improving mobility.

Here we will be discussing about the single class and multi class versions of two general types of traffic flow models, namely:

1) First order models
2) Higher order models

First order models have only a single partial differential equation, while higher order models have two first-order partial differential equations. Modelling using multiple classes was introduced by researchers in order to represent, the heterogeneity in vehicles. The partial differential equations, equilibrium flow density relationship and equation ‘flow = density*speed’ makes up the base of all these models.

2. **SINGLE CLASS MODELS**

Single class models assume that the traffic consists of only same type of vehicles and these vehicles behave exactly in a same manner under various conditions.

2.1 **First order models**

Lighthill, Whitham (1955) used the theory of kinematic waves from fluid dynamics in analysing traffic dynamics. Richards (1956) stated a simple graph shearing process for following the development of traffic waves with time and frequent appearance of shock waves. Thus a relationship was proposed between density and flow for traffic on crowded arterial road with experimental backing.

\[
\frac{\partial k(x,t)}{\partial t} + \frac{\partial q(k(x,t))}{\partial x} = 0
\]

\(k(x, t)\) density of vehicles at any distance (x), time (t)
\(q(k(x,t))\) flow at any distance (x), time (t) expressed as a function of density

This is a first-order, nonlinear PDE based on conservation of traffic flow i.e the increment of vehicles in a section is equal to the difference between upstream in-flux and down-stream out-flux in unit time. LWR model was developed for equilibrium link flow. Traffic flows are in equilibrium when the travel speed of these flows is uniquely determined as a function of traffic density, otherwise they are in non-equilibrium. LWR theory in its original form had certain deficiencies like infinite deceleration for vehicle across shockwave, assumption of equilibrium relationship for non equilibrium traffic, non representation of stop and go behaviour and platoon dispersion.

2.2 **Higher order models**

Payne (1971) suggested one of the first higher order non-equilibrium traffic models. It is written as

\[
\frac{\partial k(x,t)}{\partial t} + k(x,t)\frac{\partial v}{\partial x} + v \frac{\partial k(x,t)}{\partial x} = 0
\]

\[
\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} + a_0^2 \left( \frac{\partial k(x,t)}{\partial x} \right) / k(x,t) = (v(k(x,t)) - v) / \mu
\]
The traffic sound speed $a_0 > 0$ was constant and $v(k)$ was the relationship between speed $v$ and density $k(x,t)$ for equilibrium states. The fundamental diagram $f(k(x,t)) = k(x,t)v(k(x,t))$, reflected basic features of a roadway and $\mu$ was the relaxation time. In this model the momentum equation was derived from a car following model. This model overcame some of the deficiencies in LWR model like infinite deceleration for vehicle across shockwave, assumption of equilibrium relationship for non equilibrium traffic, non representation of stop and go behaviour, but had certain inherent deficiencies. This model did not remove all the shocks, and here vehicle could adjust their speed in response to the disturbance reaching them from behind.

Daganzo (1995) described the logical flaws in the arguments that were advanced to derive higher order continuum models, and showed that the proposed high order modifications lead to a fundamentally flawed model structure. Any continuum model of traffic flow that smoothes out all discontinuities (higher order) in density will predict negative flows and negative speeds (i.e., “wrong way travel”). He stated the reason for this as three essential differences between traffic and fluids:
1. A fluid particle responds to stimuli from the front and from behind, but a car is an anisotropic particle that mostly responds to frontal stimuli,
2. The width of a traffic shock only encompasses a few vehicles, and
3. Unlike molecules, vehicles have personalities (e.g., aggressive and timid) that remain unchanged by motion.

The critique for higher order models were given as follows
1. The PW equations were second order approximation of car following equations in spacing and speed of line of cars. They neglected 2nd, 3rd, and higher derivatives of spacing and speed which will not be small if spacing and velocity are not slowly varying.
2. Terms borrowed from the kinetic theory of gases such as relaxation time and from gas dynamics such as viscosity effects ignores the special nature of traffic particles and lead to strange prediction. The interaction in a model should not change personality of any vehicle. It means that a slow car should virtually be unaffected by interaction from a fast car passing it or queuing behind it.
3. Smoothness of the shock is inherently unreasonable because spacing and density must change abruptly whenever the road behind is empty.
4. Relaxation mechanism in a higher order model was unrealistic since it implies that desired speed distribution is a property of road and not the drivers.
5. Higher order models always exhibit one characteristic speed greater than macroscopic fluid velocity which means that future condition of a traffic stream will be determined by what is happening behind it.
6. Model that smoothes out discontinuities should some time predict negative velocities.

He also proposed that improved description of the shock structure should be based on a correct analysis of a suitable car-following model (or other microscopic model) and not on conjectures borrowed from other fields (especially continuous fluid models).

Liu et al (1998) proposed an improved higher order model which eliminated certain deficiencies in PW model. The improved model did not result in negative speeds at the tail of congested region and disturbance propagation speed greater than the traffic flow velocity. The model produced contact shocks (different speed moving at the same speed) and was able to describe the amplification of small disturbance on heavy traffic. They also took relaxation time as a function of density to incorporate real life traffic features.

The equation was given as follows
\[
\begin{align*}
\frac{\partial k}{\partial t} + \frac{\partial q}{\partial x} &= 0 \\
\frac{\partial v}{\partial t} &= \frac{1}{\mu(k)} \left[ v(k) - v \right]
\end{align*}
\]

$k, q, v$ density, flow and velocity
$v(k)$ equilibrium speed-density relationship
$\mu(k)$ relaxation time as a function of density

This relaxation term was introduced to take care of Daganzo’s critique that relaxation time was represented as a characteristic of road.

Aw and Rascle (2000) proposed a satisfactory momentum equation in order to avoid drawbacks of PW model. They proposed a momentum equation
\[
\partial_t v + (v \cdot k)p(k) \partial_x v = 0
\]
But this model was developed theoretically by introducing a velocity dependent pressure term and had an inconsistency that during solution of Riemann problem if the cars in front are much faster than the cars behind, the rarefaction waves between upstream state and intermediate state with intermediate state in a vacuum will produce a speed that will depend on initial state rather than producing the maximal possible speed. Later Zhang in his model developed $p(k)$ as a speed function $v(k)$ from a car following model.


$$\partial_t v + \left(\frac{1}{\zeta} \right) \left( v \cdot k p'(k) \right) \partial_x v = 0$$

**$\zeta$ relaxation term**

Colombo (2002) proposed a model with non linear hyperbolic conservation laws generating a Cauchy problem which is well posed for all reasonable initial data. They also considered a speed limit. The Cauchy problem admits a solution which is unique and depends continuously on the initial data. The equation was given as

\[
\frac{\partial k}{\partial t} + \frac{\partial (k,v)}{\partial x} = 0
\]

\[
\frac{\partial q}{\partial t} + \frac{\partial ((q - q_0) v)}{\partial x} = 0
\]

$k = k(t,x)$ car density  
$v = v(k,q)$ car speed  
$q = q(t,x)$ auxiliary variable in analogy with linear momentum in gas dynamics  
$k_m, q_0$ parameter characteristic of road under consideration  
$v(k,q) = \frac{1}{k} - \frac{1}{k_m}$

Jiang et al (2002) developed a new model based on a car following model. In this model the density gradient which was considered to be a main reason for unrealistic behaviour of higher order models were replaced by concept of a speed gradient. It produced a model which solved the problem of negative speed and characteristic waves travelling faster than traffic speed in other higher order models. But it had all the advantages of other higher order models. The car following model used was

\[
\partial V_{n,t}(t)/\partial t + s [V_{n,t}(\Delta x_{n,t}) - V_{n,t+1}] + \lambda (V_{n,t}V_{n,t+1})
\]

Where  
$V_{n,t}, \Delta x_{n,t}$ speed and headway respectively of following vehicle $n+1$.  
$s, \lambda$ reactive coefficients  
$V_{n,t}(\Delta x_{n,t})$ optimal speed of following vehicle ‘$n+1’$  
The macroscopic model was developed from the car following model as

\[
\frac{\partial k}{\partial t} + \frac{\partial (kv)}{\partial x} = 0
\]

\[
\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} = (v(k) \cdot v)/T + c_j \frac{\partial v}{\partial x}
\]

$c_j$ Speed of a backward propagated disturbance  
$v(k)$ equilibrium velocity density relationship  
$\partial v/\partial x$ speed gradient
3 MULTI-CLASS MODELS

3.1 First order models

Zhang and Jin (2002) proposed a model for mixed traffic where each vehicle class in the model travelled with the same group velocity. A study of such a model with two vehicle classes showed that when both classes of the traffic have identical free flow speeds, the model satisfied the first-in-first-out rule, anisotropic nature, and the usual shock and expansion waves along with family of contact waves. The conservation equation was used here as

\[(k_i)_t + (k_i v_i)_x = 0\]

Where

\[i \quad 1...N\] represent the number of vehicle classes
\[k_i(x, t)\] Concentration
\[v_i(x, t)\] Speed

The equilibrium relationship was taken here for individual vehicle class as

\[v_i = V_i(k_1, k_2, ..., k_n)\]

Where

\[v_i = V = \text{average speed of traffic stream in unrestricted region}\]

\[= \frac{k_1 v_1 + k_2 v_2}{k_1 + k_2}\]

if \[(l_1 + \zeta_1 v_1) k_1 + (l_2 + \zeta_2 v_2) k_2 < 1\]

At the restricted region we assume that vehicles are moving with a common velocity

\[V = \frac{1 - k_1 l_1 + k_2 l_2}{k_1 r_1 + k_2 r_2}\]

if \[(l_1 + \zeta_1 v_1) k_1 + (l_2 + \zeta_2 v_2) k_2 \geq 1\]

\[v_1\] and \[v_2\] free flow speed of both type vehicles
\[l_1\] and \[l_2\] effective vehicle lengths
\[\zeta_1\] and \[\zeta_2\] response time of both vehicle classes

The capacity of mixed flow depends on vehicle composition

Here in this model \[v_1 = v_2 = v_f\]

If \[k_2/k_1 = p = \text{vehicle composition}\], then critical densities of proposed speed density relations are

\[k_{1c} = \frac{1}{(l_1 + p l_2) + v_f (\zeta_1 + p \zeta_2)}\]

\[k_{2c} = p \cdot k_{1c}\]

The capacity of flow is \[v_f/(l_2 + v_f \zeta_2)\] and \[v_f/(l_1 + v_f \zeta_1)\]

Even though this model was developed to study the entrapment of low performance vehicle by fast vehicles, they just considered the perturbation on flow caused by the presence of vehicles that occupy bigger space and cause a lesser jam density. It does not consider obstruction caused by slower vehicle, overtaking of slow vehicles by fast vehicles nor psychological effects of drivers.

Wong and Wong (2002) proposed a model which grouped the heterogeneous users into nine different classes characterized by the free-flowing speeds of 60.0, 67.5, 75.0, . . , 120.0 km/h, respectively. The conservation equation for each vehicle class was taken as

\[\frac{\partial k_i}{\partial t} + \frac{\partial (k_i v_i)}{\partial x} = 0\]

\[i \quad 1...N\] represents the number of vehicle classes

Equilibrium relationship was taken separate for each vehicle class considered

\[v_i(x, t) = V_i(k_1, k_2, ..., k_n) = V_i(k_{in}) = v_f \exp \left(-(k/k_0)^2 / 2\right)\]
Where
\[ k_i, u_i \] density, speed and flow of ‘i’ class vehicles.
\[ k_{tot} \] total density of all vehicle class
\[ v_f \] free-flowing speed of user class ‘m’ on an empty highway,
\[ 'k' \] is the total density
\[ 'k_0' \] is a common parameter for all user classes

From their numerical results they were able to replicate the reverse-lambda and hysteresis and also platoon dispersion by means of multi-class modelling. It showed that discontinuity in fundamental diagram (two-capacity phenomenon) might not necessarily be caused by separate operational regimes, but is a result composition and interactions among different user groups. But the assumption in this model that the critical density was same for each vehicle gave solution which was less general.

Logghe and Immers (2003) presented a dynamic traffic flow model (LWR) for multi classes. In this model each vehicle class was described by separate fundamental diagrams which were similar. Scaling was then carried out for each vehicle class using a factor which had correspondence with passenger car equivalents. The vehicles obeyed first in first out rule. The equilibrium relationship used here was

\[ q = k_i/k_{tot} \cdot Q(k_{tot}) \]

where
\[ Q(k_{tot}) \] fundamental diagram for the total density of vehicles with respect to the reference class.
\[ k_i \] individual vehicle density
\[ k_{tot} = k_1 + r.k_2 \] for 2-vehicle classes \( r = \text{scaling factor} \)

The scale factor used here gave effects of difference in vehicle characteristics, difference in driver characteristics and difference in road characteristics. It failed to describe difference in free flow velocities, difference in speed and difference in acceleration and reaction time. But By finding this scale factor on an hourly basis this deficiencies could be overcome.

Chanut and Buisson (2003) developed a multi-class model considering 2-vehicle class’s cars and trucks. Here the equilibrium relationship was developed between individual vehicle speeds and total density. It had 2-relationships for both unrestricted and restricted regime. It predicted a slower speed of trucks at free flow regime and equal speed for both vehicle classes at congested regime. At the congested region the flow was assumed in passenger car equivalents ie) a linear decrease of flow was assumed.

For free flow regime the relation was given as

\[ v_1 = V_{f1} - (V_{f1} - V_c)(k_1 + k_2)/k_1(k_1,k_2) \] (for cars)
\[ v_2 = V_{f2} - (V_{f2} - V_c)(k_1 + k_2)/k_2(k_1,k_2) \] (for trucks)

Where
\[ V_{f1} \] free flow speed of individual classes
\[ k_1 \] density of individual classes
\[ k_c(k_1,k_2) \] critical density for total vehicle classes
\[ v_c \] speed at critical density for total vehicle classes

For congested regime

\[ v_i = q_{pec} / (k_1 + e.k_2) \]
\[ q_{pec} = \text{total equivalent flow} = \frac{\epsilon(k_{jam}(k_1,k_2)-(k_1+k_2))}{k_{jam}(k_1,k_2)-k_2(k_1,k_2)} \]
\[ k_{jam}(k_1,k_2) = \frac{N(k_1+k_2)}{k_1+k_2} \]
\[ k_c(k_1,k_2) = k_{jam}(k_1,k_2) * \mu \]
\[ e \] passenger car equivalent for truck = \( L_2/L_1 \)
\[ L_2, L_1 \] Lengths occupied by cars and trucks when they are stationary in jam.
\[ \mu \] Experimental term in the range of 0.2 to 0.5.

Even though they were able to model overtaking phenomena and effect of the length of each vehicle class on other vehicle class’s speed, they failed to describe the entrapment of high speed vehicles by slower vehicles.

Ngoduy and Liu (2007) proposed a multi-class first-order simulation model in order to represent non-linear phenomena such as platoon dispersion, hysteresis and capacity drop. In the developed model, each vehicle class
is only characterized by their desired speeds in a free-flow traffic state where overtaking is allowed. However, when traffic is congested, all vehicle classes must travel at the same congested speed and overtaking is not possible. The velocity in the free flow region was taken as

\[
V_i = \frac{r}{r_{cr}} V_{cr} + \left(1 - \frac{r}{r_{cr}}\right) V_{if} \quad \text{(In free-flow state, i.e. } r < r_{cr})
\]

where \( r \) denotes the total density of all classes

\[
r = \sum_i r_i.
\]

\( V_{if}, r_{cr} \) are model parameters that depict, respectively, the class specific free speed, the weighted average of the class specific critical density

\( V_{cr} \) is the critical speed, determined as: \( V_{cr} = \frac{q_{cap}}{r_{cr}} \) with \( q_{cap} \) being the weighted average of the class specific capacity.

The velocity in congested region was taken as

\[
V_i = V = \frac{r_{jam} - r}{r_{jam} - r_{cr}} V_{cr} r_{cr} = \frac{r_{jam} - r}{r_{jam} - r_{cr}} q_{cap} \quad \text{(In congested state i.e. } r \geq r_{cr})
\]

They assumed a share factor, which denotes the fraction of the number of vehicle class ‘i’ over the total number of vehicles in cell ‘z’.

\[
a_i^z = \frac{r_i}{r_z}
\]

The weighted average parameters of our fundamental diagram are determined for every cell ‘z’ as

\[
r_{z,cr} = r_{cr}^0 \sum_i \frac{a_i^z}{\beta_i^0}, \ q_{z,cap} = q_{cap}^0 \sum_i \frac{a_i^z}{\beta_i^0}, \ r_{z,jam} = r_{jam}^0 \sum_i \frac{a_i^z}{\beta_i^0}
\]

Where \( r_{cr}^0, r_{jam}^0 \) and \( q_{cap}^0 \) denote, respectively, the critical density, jam density and capacity of a reference class, which are determined in passenger car units (pcu). \( \beta_i \) denotes the PCU factor of vehicle class \( i \), which represents the differences in the amount of interference to other traffic in mixed traffic operations.

Logghe and Immers(2008) proposed a new model where classes interact on a non-cooperative way. Here the homogenous vehicle class relation was taken as triangular and there existed 3-regions: free flow regime, congested regime and semi congested regime in the equilibrium relationship. In the free flow regime vehicle classes moved with their free flow velocity without interrupting each other. In semi congested region the fast vehicles are in congested region and would reduce their speed but not less than speed of slow vehicles. Slow vehicles moved with their free flow speed. In congested region both vehicles moved with same velocity. This leads to anisotropic behaviour of the traffic stream. This means that vehicles only react on stimuli in front of them. But the model completely depended in equilibrium relationship.

In free flow region phase boundary is given as

\[
k_1 + k_2 \leq k_{M1}
\]

In semi congested region speed of faster class vehicles are given by

\[
v = k_{M1, M2} \frac{k_1 k_{M2} - k_{M2} k_{j1} + k_2 k_{j1}}{(k_1 k_{M2}) k_{M1, M2} - k_1 k_{M2} k_{j1}}
\]

The phase boundary is derived as \( \frac{k_1}{r k_{M2}} + \frac{k_2}{r_{M2}} \leq 1 \)

In congested region the speed of both vehicle classes are derived as

\[
v = \frac{k_1/k_{j1} + k_2/k_{j2} - 1}{w_1 k_{j1} + w_2 k_{j2}}
\]

Here

\( k_i \) individual density of vehicle classes

\( k_{mi} \) individual critical density of vehicle classes

\( k_{ji} \) individual jam density of vehicle classes

\( w_i \) congestion wave speed
This model explained the overtaking in free flow region, obstruction of fast vehicles in the semi congested region, along with the satisfaction of anisotropy. But in free flow region this model slightly over estimated the actual flow since it has assumed that both the vehicle classes are free from interactions.

### 3.2 Higher order models

Tang et al (2009) developed a non-equilibrium higher order macroscopic model with this disturbance propagation speed concept for heterogeneous traffic for single lane. It was a speed gradient model. They considered 2-vehicle class’s car and buses. At first they developed a microscopic model assuming that a ‘j’ class vehicle will always led an ‘i’ class vehicle. This model considered the multiple speed as well as the effects of each class on car following behaviour of heterogeneous traffic flow. The model was as follows

\[
\frac{\partial V_{i,a}(t)}{\partial t} + s_i \left( V_{i,a}(\Delta x_{i,a}) - V_{i,a} \right) + \sum_{j=1}^{N} \lambda_{ij} p_{ij} (V_{j,a} - V_{i,a})
\]

Where

- \( V_{i,a}, \Delta x_{i,a} \) are respectively speed and headway of vehicle i,n.
- \( s_i, \lambda_{ij} \) are reactive coefficients
- \( N \) = Number of classes
- \( V_{i,a}(\Delta x_{i,a}) \) = optimal speed of vehicle i, n

From that a macroscopic model was formed

\[
\frac{\partial k_i}{\partial t} + \frac{\partial (k_i v_i)}{\partial x} = 0 \quad \text{(Continuity equation)}
\]

\[
\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial x} = \frac{v_{ie}(k)}{T_i} + \sum_{j=1}^{N} \left( \frac{c_j k_j(x, t)}{k(x, t)} \cdot \frac{\partial v_j}{\partial x} + \frac{1}{\tau_{ij}} \frac{k_j(x, t)}{k(x, t)} (v_j - v_i) \right)
\]

(Momentum equation)

Where

- \( i, j \) = Following and leading vehicle classes respectively
- \( v_{ie}(k) \) = Equilibrium speed of ‘i’ class
- \( T_i, \tau_{ij} \) = Adjustment and anticipation times of ‘i’ class
- \( c_j \) = Characteristic wave velocity at jam density for ‘j’th vehicle class
- \( k = \sum_{i=1}^{N} k_i \) = Total density
- \( k_i(x, t), k_j(x, t) \) = ‘i’ and ‘j’ class density at point x, t respectively
- \( v_i(x, t), v_j(x, t) \) = ‘i’ and ‘j’ class speed at point x, t respectively
- \( N \) = Total number of vehicle classes

The equilibrium velocity density relationship used in this model was proposed by Del Castillo and Benitez (1995).

\[
V = V_{if}(1 - \exp \left( \frac{1 - \exp \left( \frac{C_j}{V_{if} K - 1} \right)}{K} \right))
\]

- \( C_j \) = Characteristic wave speed under the jam density of each vehicle class
- \( K_j \) = Total jam density
- \( K \) = Total density
- \( V_{if} \) = Free flow velocity of each vehicle class

The jam density is found out using formula

\[
k_{jam} = \frac{1}{2 \sum_{i=1}^{N} p_{ri} (L_i + h_i)}
\]

where

- \( k_{jam}, L_i, h_i, p_{ri} \) are respectively the jam density, length of vehicle, safe headway of vehicle and probability associated with leading vehicle.
4. Conclusion

A traffic flow model is an abstract of traffic flow phenomenon. It represents the salient properties of traffic flow like driver’s behavior, vehicle type and interaction with other vehicles. Modelling could be done on a sub-microscopic scale, microscopic scale, mesoscopic scale or a macroscopic scale, but a macroscopic scale modeling is more preferred due to the fewer amounts of details involved. The parameters used in macroscopic modeling are density, flow and speed. In macroscopic modeling we try to predict the future characteristics of traffic along a stretch of road for small time increments. It is done by representing the propagation of model parameters in form of partial differential equations. It is very helpful in understanding the formation and propagation of traffic congestion. Hence, traffic researchers use traffic flow models to identify possible bottlenecks. It can also be used to simulate the conditions after implementation of different strategies for improving mobility.

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